UNIT 6:
NUCLEAR THEORY

"When I was young I used to feel so alive, so dangerous! In fact, would you believe I started life as a krypton-238? Then one day I accidentally ejected an alpha particle, now look at me, an spent old atom of Lend-206; Seems that all my life since then has been nothing but decay, decay, decay..."
A. NUCLEAR PHYSICS

Al. Atomic Nuclei

Using the scattering experiment (1911 - 1913), Rutherford and others showed that the nucleus was composed of protons (p+) and neutrons (n). These quantized nuclear particles are called nucleons.

Thus, there are three parts to the atom:

<table>
<thead>
<tr>
<th></th>
<th>Charge (e)</th>
<th>Mass (u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrons</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>Protons</td>
<td>+1</td>
<td>1</td>
</tr>
<tr>
<td>Neutrons</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Nuclear Notation

To fully describe the nucleus of an atom (e.g. Sodium-25), we use the following notation:

$$\text{mass number} \quad \text{atomic number} \quad \text{charge of nucleus (in e)}$$

Isotopes

- isotopes have the same number of protons (same element), but different numbers of neutrons
- they have the same chemical properties, thus they can form the same compounds
- however, they can have different nuclear properties (some are stable, while others are radioactive)

Strong Nuclear Force

- the protons in the nucleus are very close together
- since they have like charges, there is a strong repulsion force (high Fe)
- there must be another force that holds the nucleus together, called the strong nuclear force

- the strong nuclear force has the following properties:
  - it is a force of attraction that acts between any pair of nucleons (e.g. n-n, n-p, p-p)
  - it is a "short force"
- when the distances are very short (10^{-15} m), it is the strongest force in nature
- but when the distance increases, this force decreases dramatically

Stability

- there are only 400 isotopes that are stable (not radioactive)
- small stable atoms (less than 20 protons) typically have equal numbers of protons and neutrons
- however, larger stable atoms (> 20 protons) have many more neutrons than protons

Nuclear Mass Defect

The total mass of a stable nucleus is always less than the sum of the masses of its constituent protons and neutrons. The difference in mass is called the mass defect ($\Delta m$) of the nucleus. To calculate the mass defect, you can use the formula:

$$\Delta m = (\text{Total mass of protons and neutrons}) - (\text{Actual nuclear mass})$$

Mass-Energy Equivalence

In 1905, Einstein proposed that mass was another form of energy. Thus, the mass that was "lost" in the nucleus when it was first constructed was converted to another form of energy. To calculate the equivalent energy from the mass defect, Einstein introduced his most famous formula:

$$E = \Delta mc^2$$

where

- $\Delta m$ is the mass defect (in kg)
- $c$ is the speed of light (in m/s)
- $E$ is the energy converted from mass (in J)

As a result of this, physicists introduced a more general conservation law: the law of conservation of mass-energy.

Note: Subatomic masses are often expressed in terms of the electron-volt energy unit: $eV/c^2$.

To illustrate, consider the conversion of 1 atomic mass units to $MeV/c^2$:

First, convert to kg:

$$\Delta m = 1 u = 1.66 \times 10^{-27} \text{ kg}$$

Next, use Einstein's formula to convert this mass to energy:

$$E = \Delta mc^2 = (1.66 \times 10^{-27} \text{ kg}) (3.00 \times 10^8 \text{ m/s})^2 = 1.494 \times 10^{-10} \text{ J}$$

Finally, convert to the desired unit:

$$m = \frac{E}{c^2} = \frac{934 MeV}{c^2}$$

Binding Energy

The equivalent energy associated with the mass defect of the nucleus is called the binding energy of the nucleus. The binding energy represents the minimum energy that must be put into the nucleus in order to completely break it apart into its component protons and neutrons. It is also the energy released when the protons and neutrons come together to form an atom.

It is equivalent to the ionization energy of an electron for an atom. For example, in the Bohr model of the hydrogen atom, it takes 13.6 eV of energy to raise the electron from ground state to $n = \infty$, thus removing the electron from the atom- so, 13.6 eV is the electron's binding energy. If a proton captures an electron to make a hydrogen atom, then energy is released (as photon energy) when the electron descends to ground state.

In fact, it is this binding energy which keeps the atom stable. If the mass of the nucleons was equal to the mass of the nucleus, then it would immediately fall apart with no energy added at all.
**HOMEWORK** (The Nucleus)

A.  1. For the following isotopes, determine:
   - the nuclear notation
   - the nuclear charge (in C)
   - the number of neutrons, protons, and nucleons
   - the total mass of the constituent nucleons (in kg)
   
     a) Cadmium-114   (Cd-rr4)  
     b) Thallium-205   (Tl-205)

2. a) How much mass (in kg) must be converted to generate 7.0 GeV of energy?
   b) How much energy (in keV) is generated from 1.5 x 10^{-30} kg of mass?

3. If the nuclear mass defect for an isotope is 0.095298 u, determine the binding energy (in MeV). Answer to 1 decimal place.

4. The actual nuclear mass of Gallium-69 is 68.925580 u. Determine:
   a) the nuclear mass defect (in kg)
   b) the nuclear binding energy (in MeV) Answer to 1 decimal place.

B.  5. Identify the isotope from the following nuclear descriptions.
   a) 70 protons and 904 neutrons  
   b) 50 neutrons and 90 nucleons
   c) nuclear charge is 1.056 x 10^{-17} C and the total neutron mass is 1.6366 x 10^{-25} kg

6. Convert the following to the desired unit.
   a)  1.6 x 10^{-12} J; m = 1.26 x 10^{-25} kg
   b)  1.359 x 10^{-25} kg
   c)  1.8 x 10^{-29} kg

7. The nuclear binding energy for an isotope is 448 MeV. Determine its nuclear mass defect (in u).

8. The binding energy for Fluorine-19 is 8.993 MeV. Determine its nuclear mass, in atomic mass units. Answer to 3 decimal places.

**SOLUTIONS**

1. a) $^{48}$Pd ; 48 p, 66 n, 114 nucleons ; $Z = 7.68 \times 10^{-18}$ C ; $m = 1.90 \times 10^{-25}$ kg
   b) $^{53}$Zr/ ; 81 p, 24 n, 205 nucleons ; $Z = 1.30 \times 10^{-17}$ C ; $m = 3.42 \times 10^{-25}$ kg

2. a) $E = 1.136 \times 10^8$ J ; $m = 1.26 \times 10^{-25}$ kg
   b) $E = 1.359 \times 10^{-13}$ J = 8.49 x ro^{-5} eV = 849 keV

3. $m = 1.582 \times 10^{-28}$ kg ; $E = 1.424 \times 10^{-11}$ J = 89.0 MeV

4. a) $E = 1.44 \times 10^{-25}$ kg ; $m = 1.8 \times 10^{-29}$ kg
   b) $E = 5.985 \times 10^{-10}$ J = 3.74 GeV/c^2
   c) $E = 2.224 \times 10^{-11}$ J ; $m = 2.47 \times 10^{-28}$ kg = 0.149u

5. a) Yb-rr4  
   b) Zr-90
   c) $m = 66.98$ n ; Dy-rr4

6. a) $E = 7.168 \times 10^{-11}$ J ; $m = 7.9644 \times 10^{-28}$ kg = 0.480u
   b) $E = 5.985 \times 10^{-10}$ J = 3.74 GeV/c^2
   c) $E = 2.224 \times 10^{-11}$ J ; $m = 2.47 \times 10^{-28}$ kg ; $m = 3.529 \times 10^{-26}$ kg = 0.149u

p.25
A3. Radiation

Types of Radiation
- when nuclei are unstable, they are radioactive and emit radiation
-it was discovered that there were three types of radiation emitted by radioisotopes:

1. Alpha particles \( (\alpha^2) \)
   - these are Helium-4 nuclei and they have the lowest penetration ability
     ----+ blocked by skin, a sheet of paper, or a few em of air
   - dangerous when swallowed, since they will ionize the cells of the digestive tract
     (and possibly cause cancer to internal surface cells)

2. Beta Particles (-)
   - these are high energy electrons and they have more penetration ability
     ----+ penetrate skin, but blocked by many sheets of paper, wood, a few mm of aluminum,
     - dangerous to outside skin cells (like UV radiation), but also dangerous when swallowed
       (however, they are not as dangerous as alpha particles, since they ionize less frequently)

3. Gamma Radiation \( (\gamma) \)
   - these are very high frequency, high energy photons and they have the greatest penetration
     ----+ they pass right through the body, but are blocked by several em of lead, a couple
     - since gamma radiation can penetrate through the entire human body, they can ionize any
       cells (including inside the bones) ; however, they tend to pass through the body without
       ionizing any cells, so they are dangerous only at high intensities and for long time periods

- these 3 types of radiation can be separated using magnetic or electric fields

Note:
We are constantly being bombarded by low amounts of background radiation from the sun, from
radioactive material stored in brick, and from radioactive material underground. Low intensity
radiation is unlikely to do lasting damage to our body - damaged cells are simply repaired.
However, radiation is dangerous if we are exposed to: (1) a huge, short-term dose of radiation, and
the resulting massive ionization causes radiation sickness and likely death, or (2) a moderate, steady
dose of radiation, which often leads to cancer.
Three Decay Reactions

When you balance nuclear equations:

Top numbers obey the law of conservation of nucleons (mass number)
Bottom numbers obey the law of conservation of charge

1. Alpha Decay
- nucleus has too many protons; the electric force of repulsion is too high for the nucleus to be stable
- the nucleus emits an alpha particle (He-4) to reduce the size and charge of the nucleus

   The nucleus loses 2 protons and 2 neutrons.
   Thus, the nuclear charge goes down by 2, while the nuclear mass goes down by 4

   e.g. Alpha decay of Radon - 222:

   \[
   ^{222}\text{Rn} \rightarrow ^{4}\text{He} + ^{218}\text{Po} \quad (+\gamma)
   \]

   Note: Energy is released from the mass defect, where \(11m = m_{\text{parent}} - m_{\text{daughter}} - m_a\)
   This energy is in the form of the kinetic energy of the products (as well as the energy of the gamma photon, if one is emitted).
   This reaction must also obey the law of conservation of momentum.
   - since the number of protons change, the element changes (transmutation)

2. Beta Decay (also called Beta-Negative Decay)
- the nucleus has too many neutrons to be stable
- due to the weak nuclear force, a neutron changes into a proton and emits a high energy electron (p^+)

   This reduces the number of neutrons by 1, but increases the number of protons by 1
   Thus, the nuclear charge goes up by 1, while the total nuclear mass stays the same

   e.g. Beta Decay of Sodium - 24:

   \[
   ^{\frac{1}{2}}\text{Na} \rightarrow ^{\frac{1}{2}}\text{Mg} + ^{\frac{1}{2}}\nu \quad (+ \ r)
   \]
   Energy is released due to mass defect.
   where \(\nu\) is an anti-neutrino

Note: When the scientists first studied beta decay, the daughter nucleus and the emitted beta particle did not account for all of the energy or momentum. This was a huge concern. So, in order to ensure that the laws of conservation of energy and momentum were satisfied, Pauli (1930) introduced a new particle (which was later to be called a neutrino by Fermi) that had no mass or charge but had the remaining energy. Neutrinos are created inside the sun and they are so tiny that they can travel right through the Earth with little probability of interaction!

Beta decay, much like alpha decay, is a transmutation reaction.
2b. Beta-Positive Decay (also called Beta Decay - Positron Emission)

This is a second type of beta decay, where a proton turns into a neutron (again, due to the weak nuclear force), and as a result, a positron and a neutrino are emitted.

e.g. 
\[ ^{12}_8 O \rightarrow ^0_1 \beta + ^{15}_7 N + ^0_0 \nu \quad \left( + \gamma \right) \]

Energy is released due to the mass defect.

where 
\( p \) is a positron (the anti-particle of an electron - discussed later)
\( \nu \) is a neutrino

3. Gamma Decay

During alpha and beta decay, the nucleus can be left in an excited state. Since the excited state is unstable, it will return to nuclear ground state. In so doing, it will emit a gamma photon of very high energy. Since there is no transmutation involved, this reaction can be written as

\[ ^A_X \cdot ^A_Z + r \]

Thus, when a radioisotope goes through alpha decay (for example), there are two possibilities:
(1) only an alpha particle is emitted with very high energy, or
(2) an alpha particle is emitted with lower energy, but the daughter nucleus is in an excited state, and when it drops to ground state, it will emit a gamma ray

Decay Series

When a radioisotope goes through decay (either alpha or beta), the daughter nucleus is often unstable as well. So, it will go through decay as well, giving rise to another daughter nucleus. This can happen many times, which leads to a series of decays. These decays can be summarized on a special graph. One decay series for uranium-238 is shown below:

![Decay Series for U-238](image-url)
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**Unit 6: Nuclear Theory**

**HOMEWORK**  (Nuclear Decay)

Both A and B

1. Nuclear radiation is sent through a perpendicular magnetic field directed out of the page, as shown. Predict the path of each type of radiation.

2. Write complete, balanced equations for each of the following decay reactions.
   a) the alpha decay of americium-243 (Am-243)
   b) the beta-negative decay of argon-39 (Ar-39)
   c) the beta-positive decay (positron-emission) of fluorine-19 (F-19)
   d) the alpha decay of a parent isotope produces a daughter nucleus of thorium-231 (Th-231)
   e) the beta-negative decay of a parent isotope produces a daughter nucleus Xe-131
   f) lead-212 (Pb-212) decays to bismuth
   g) plutonium-239 decays to uranium
   h) Carbon-10 (C-10) decays to boron (B)

A. 3. A stationary Po-214 decays to Pb-210 (m = 209.98418 u) and emits only an alpha particle at a speed of $1.48 \times 10^7$ m/s. Determine:
   a) the speed of the Pb-210 nucleus
   b) the mass defect

4. In a beta-positive decay reaction, the mass defect is $5.81 \times 10^{-29}$ kg. If the beta particle and the neutrino both have 13.0 MeV of energy, and a gamma ray photon is also emitted, then determine the maximum frequency of the gamma ray photon.
   Ignore the kinetic energies of the nuclei.

B. 6. In an alpha decay reaction, an alpha particle is emitted at a speed of $9.09 \times 10^6$ m/s and a 2.46 pm gamma photon is also emitted. If we can ignore the speeds of the parent and daughter nuclei, then determine the mass defect (in kg).

7. In one reaction, U-232 (m = 232.037131 u) decays to Th-228 (m = 228.028716 u) and emits one particle. Ignore the kinetic energies of the nuclei and assume no gamma photon is emitted.
   a) Identify the emitted particle.
   b) Determine the mass defect (in kg).
   c) Determine the maximum possible speed of the emitted particle?

8. When boron-12 decays to carbon-12, there is a mass defect of $1.435 \times 10^{-2}$ u and the emitted beta particle has an energy of 9.00 MeV. A second particle is also emitted.
   a) Identify the two particles emitted. Assume no gamma photon is emitted.
   b) Determine the maximum possible energy (in MeV) of the second particle.

p.129
9. One decay series for Neptunium-237 results in a stable Bismuth-209 nucleus in 11 steps:

\[ a, - , a, a, - , a, a, - , a, - \]

a) Show complete, balanced nuclear equations for all 11 steps.
b) Sketch a decay series graph.

SOLUTIONS

1. \( a^+ \) deflects down; - deflect upward a lot more; \( y \) travels straight through (undeflected)

2. a) \( ^{243}_{95}Am \rightarrow \frac{4}{5}a + \frac{2}{5}Np \ (+r) \)
b) \( ^{33}_{47}Ar \rightarrow \frac{2}{5}p + \frac{2}{5}K + \frac{2}{5}v \ (+r) \)
c) \( ^{99}_{43}Fr \rightarrow 10^{+} + 1O + u \ (+r) \)
d) \( ^{83}_{32}As \rightarrow \frac{4}{5}a + 2 \\ beta \ (+r) \)
e) \( ^{85}_{31}Ml \rightarrow \frac{1}{3}O + \frac{2}{3}xe + \frac{2}{3}v \ (+r) \)
f) \( ^{200}_{82}Pb \rightarrow \frac{10}{3}t \ + \frac{2}{3}B + \frac{2}{3}u \ (+r) \)
g) \( ^{236}_{94}Pu \rightarrow \frac{2}{5}a + \frac{2}{5}U \ (+r) \)
h) \( ^{1}C \rightarrow 10^{+} \ + 1B + u \ (+r) \)

3. a) Cons of \( p \): \( 0 = PPb + Pa \) \( V = 2.82 \times 10^{5} \text{ m/s} \) (opposite direction of the alpha particle)
b) Cons of m-E: \( E = E_{p} = E_{k} + E_{kg} = 7.422 \times 10^{-13} \text{ J} \) \( ; \) Using \( E = \frac{1}{2} m c^{2} \), \( \times m = 8.25 \times 10^{-30} \text{ kg} \)

4. \( EL'm = 5.229 \times 10^{-12} \text{ J} \) \( ; \) \( Er = EL'm - 2 (2.08 \times 10^{-12} \text{ J} ) = 1.069 \times 10^{-12} \text{ J} \) \( ; \) \( f = 1.61 \times 10^{21} \text{ Hz} \)  
Assumption: Ignoring the velocities of the parent and daughter nucleus.

5. \( 1^{13}Cu \rightarrow 1^{13}Nl + u \ (+r) \)

6. \( Eka = 2.75 \times 10^{-13} \text{ J} \) \( ; \) \( Er = 8.085 \times 10^{-14} \text{ J} \) \( ; \) \( EL'm = Eka + Er = 3.56 \times 10^{-13} \text{ J} \) \( ; \) \( \times m = 3.95 \times 10^{-30} \text{ kg} \)

7. a) Alpha decay, so an \( a^+ \) is emitted 
b) \( 1^{4}Jm = mu = 232 - mTh - 228 = 3.97 \times 10^{-30} \text{ kg} \)
c) Cons of m-E: \( Eka = EL'm = 3.572 \times 10^{-13} \text{ J} \) \( v = 1.04 \times 10^{7} \text{ m/s} \)

8. a) An electron and an antineutrino are emitted 
b) \( E,Jm = 2.144 \times 10^{-12} \text{ J} = 13.4 \text{ MeV} \) 
\( \text{Eu} = EL'Jm - 9.00 \text{ MeV} = 4.40 \text{ MeV} \)

9. a) \( \text{Np}-237 , \text{Pa}-233 , \text{U}-233 , \text{Th}-229 , \text{Ra}-225 , \text{Ac}-225 , \text{Fr}-221 , \text{At}-217 , \text{Bi}-213 , \text{Po}-213 , \text{Pb}-209 , \text{Bi}-209 \)
b) See diagram.
A4. Measuring Radioactivity

Radioactivity is the rate of radioactive emissions (α, β, γ) per unit time.

- Measured by a Geiger counter
  
  Units:  
  
  Bq 1 Bq = 1 emission per second

- Curie discovered that radioactivity is NOT affected by things that affect chemical reactions (e.g., temperature, pressure, bonding with other atoms, etc.).

- There are only two factors that affect radioactivity:
  
  1. Mass (Amount) of Sample
     - The more mass in the sample
       => More nuclei going through decay
       => More emissions (activity)

  2. Half-Life \( T_{1/2} \)
     - The time required for half of the atoms (mass) in a sample to decay (transmute to different atoms)

   Example: Thorium-234 has a half-life of 24 days

   ![Graph](image)

   Day 0: 100% Th-234
   Day 24: 50% other atoms, 50% Th-234
   Day 48: 75% other atoms, 25% Th-234

   Note:
   - The larger the sample and the shorter the half-life, the greater the radioactivity

Equations:

\[
N = N_0 \left( \frac{1}{2} \right)^n
\]

\[
t = \frac{\text{total time}}{T_{1/2}}
\]

where

\( N_0 \) = original amount (or activity) in percent
\( N \) = final amount (or activity) in percent
\( n \) = number of half-lives

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Must have same units

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p.130
1. The decay of lodine-131 (I-131) is shown on the graph. Using the graph alone, determine:
   a) the half-life of I-131
   b) the percent of sample remaining after 36 days
   c) the time it takes to have 90% of the sample to decay to other atoms
   d) the mass of a 24 g sample of I-131 after 4.0 days
   e) the time it takes for 300 g of I-131 to decay to 60 g

2. Strontium-90 has a half-life of 28.8 years and it goes through beta-negative decay. A 6.50 kg sample of Strontium-90 is left on the shelf for 75.0 years.
   a) Write a balanced equation for its decay.  
   b) How much Strontium-90 is left?

3. Magnesium-29 has a half-life of 9.5 minutes. A sample has been left in storage for 3.0 hours and its final activity is 460 Bq. What was its original activity?

4. A 920 g sample of Cobalt-60 has decayed to a final sample of 14.375 g. If the half-life of Cobalt-60 is 5.3 years, then how long was the sample in storage?

5. A sample of Lead-212 has been left in storage for 2.21 days, and at the end, only 3.125% of the original sample is left.
   a) What is the half-life of Lead-212? Answer in hours.
   b) If it goes through alpha decay, then write a complete balanced equation for this reaction.
   c) If the mass defect is 0.0750 u and the alpha particle is emitted at a speed of 4.60 x 10^7 m/s, determine the minimum wavelength of the gamma photon emitted.

6. A radioactive nuclide produces 2880 decays per minute at one time, and 1.6 hours later, produces 820 decays per minute. What is the half-life of the nuclide? Answer in minutes.

7. An ancient club is found with a ^14C / ^12C ratio of 4.00 x 10^{-13}. Determine its age, assuming that in living trees, the ratio of ^14C / ^12C is about 1.3 x 10^{-12}. Carbon-14 is radioactive and it decays with a half-life of 5730 years.

**SOLUTIONS**

1. a) 8.0 days  
   b) 4-5%  
   c) 10% remains ; 26 days  
   d) 70% remains 17 g of the sample is left  
   e) 20% remains ; 18 days
2. a) ^3P^+; y+g V(+r)  
   b) n = 2.6042 half-lives ; N = 1.07 kg
3. n = 18.9474 half-lives ; No= 2.3 x 10^8 Bq  
   4. n = 6 half-lives ; t = 32 years
4. a) n = 5 half-lives ; T/2 = 10.6 hours  
   b) z; Pb^+; a+^2 Hg + r
   c) E_y=ELJJn-Eka = 1.12 x 10^{-11} J -7.04 x 10^{-12} J=4.17 x 10^{-12} J ; ..l.=4.77 x 10^{-14} m
5. 53 minutes  
6. 7.97 x 10^3 years

p.132
AS. Mass Spectrometer (Determining Isotopes)
- essentially, a Thomson device that is used to determine the nature of different isotopes of an element

- **Ion Generation**
  - the isotope must be charged so that it will be affected by $E$ and $B$
  - an electron beam hits the atoms, ionizing them and giving them a positive charge

  If they are *singly ionized*, they have lost one electron ($q = +e$)
  If they are *doubly ionized*, they have lost two electrons ($q = +2e$)

- ** Ion Acceleration**
  - the ions are then accelerated through a potential difference (from rest)
  - this gives them a high speed as they enter the velocity selector

- **Velocity Selection Chamber**
  - in this chamber, there is both an electric field and a magnetic field
  - ions enter this chamber at different speeds
  - only ions with the correct speed will be able to pass through undeflected

- ** Ion separation chamber**
  - all ions that enter this chamber are travelling at the same speed $v$
  - the magnetic field causes the charges to move in circular motion
  - different isotopes (masses) will travel along different paths

- ** Ion Detector** (Collector)
  - the ions hit a photographic film, where they leave a mark (glow)
  - the greater the number of particles that hit the plate in a certain location, the darker (wider) the mark
  - in this way, not only can experimenters know the respective masses, they can also determine
    - the percentage of each type of isotope (which is important for radiocarbon dating)
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HOMEWORK (Mass Spectrometer)

A. 1. A singly-ionized argon-40 atom travels undeflected through a velocity selector, having mutually perpendicular electric and magnetic fields. If the experimenter selects a speed of 4.70 x 10^5 m/s, and the magnetic field strength is 910 mT, then find the magnitude and direction of the electric field between the plates.

2. Singly-ionized chlorine atoms enter a perpendicular magnetic field at a speed of 8.00 x 10^5 m/s. If the magnetic field is 6.80 T acting into the page, then determine:
   a) whether the path is clockwise or counterclockwise
   b) the radius of the circular paths (in cm) for:
      (i) Cl-35, if its mass is 34.969 u
      (ii) Cl-37, if its mass is 36.966 u

3. Doubly-ionized Tin-120 (119.902 u) isotopes are accelerated from rest through a potential difference of 11.0 kV. What is the final speed of these ions?

B. 4. An unexcited atom requires a minimum of 35 eV to be ionized. What is the minimum speed required by an incident electron to ionize this atom?

5. A doubly-ionized Beryllium-9 atom travels undeflected through a velocity selector, having mutually perpendicular electric and magnetic fields. If the voltage between the parallel plates is 800 V and the distance between the plates is 7.50 cm, than what is the required magnetic field to select a speed of 200 km/s? Answer in mT.

6. Singly-ionized Cerium (Ce) atoms are first sent through a velocity selector (jE_1 = 61.0 kV/m; B_1 = 210 mT). The atoms that make it through the velocity selector then enter into a perpendicular magnetic field (B_2 = 900 mT). If the radius of their circular path is 45.8 cm, then what is the mass of the isotope (in atomic mass units, u)?

7. Triply-ionized Vanadium-51 (50.944 u) isotopes are accelerated from rest through a perpendicular 750 mT magnetic field and go into circular motion with a diameter of 40.0 cm. Find the potential difference V. Answer in kV.

SOLUTIONS

1. 4.28 x 10^5 V/m ; Fm acts down, Fe acts up, and so electric field goes up (t)
2. a) Fm is up when it first enters, so counterclockwise
   b) (i) m = 5.805 x 10^26 kg ; r = 4.27 Cm (ii) m = 6.136 x 10^26 kg ; r = 4.51 em
3. q = 3.2 x 10^-19 C ; m = 1.99 x 10^25 kg ; v = 1.88 x 10^8 m/s
4. E_k = 5.6 x 10^-18 J ; v = 3.5 x 10^6 m/s
5. 5.33 x 10^-2 T = 53.3 mT
6. v = 2.904 x 10^6 m/s ; m = 2.2705 x 10^25 kg = 137 u (Ce-137)
7. m = 8.457 x 10^26 kg ; q = 4.8 x 10^-19 C ; r = 0.20 m ; v = 8.514 x 10^5 m/s \ LTV = 63.9 kV
A6. Nuclear Reactions

Artificial Transmutations

In 1919, Rutherford bombarded stable nitrogen-14 with alpha particles and caused it to transmute. I.e.

$$^{14}_7\text{N} + ^4_2\text{a} \rightarrow ^{17}_8\text{O} + ^1_1\text{H}$$

After this, scientists tried to create new elements using particle accelerators (discussed later in this chapter). All elements beyond uranium are artificial (none of which are stable).

Fission Reactions (discovered by Fermi, 1938)

- when a large nucleus splits into two smaller nuclei
- triggered by a slow-moving (low energy) neutron

E.g. Fission of Uranium-235:

$$\frac{235}{92} \text{U} + \frac{1}{1} \text{n} \rightarrow \frac{141}{56} \text{Ba} + \frac{92}{36} \text{Kr} + 3\frac{1}{0} \text{n} + \gamma$$

Energy from the mass defect is converted to:

1. $E_k$ of neutrons and nuclei
2. $E$ of gamma photon(s)

The 3 neutrons released can trigger new reactions (chain reaction).

If controlled, then it is a reactor. If it is uncontrolled, it is a bomb.

Advantages:
- Easy to trigger
  - needs only low energy neutrons
- Higher energy yield than coal
  - 2 million times more!

Disadvantages:
- Radioactive waste
  - expensive to dispose of
- Nonrenewable resource

Fusion Reactions

- when small nuclei fuse together and make a larger nucleus
- these reactions occur naturally in the sun

E.g. Fusion of Deuterium and Tritium:

$$\frac{2}{1} \text{H} + \frac{3}{1} \text{H} \rightarrow \frac{1}{0} \text{n} + \frac{4}{2} \text{He} + \text{Energy}$$

Energy from the mass defect is converted to:

1. $E_k$ of neutrons and nucleus
2. $E$ of gamma photon(s)

Advantages:
- abundant fuel
- stable by-products

Disadvantages:
- high temperatures ($1 \times 10^8$ K!) are required to overcome repulsion of nuclei
**HOMEWORK** (Nuclear Reactions)

1. Using balanced nuclear equations, answer the following:
   a) Aluminum-27 (Al-27) is bombarded by an alpha particle. As a result, a proton is emitted and a new daughter nucleus is formed. What is the daughter nucleus?
   
   b) Beryllium-9 (Be-9) is bombarded by a high-speed proton. If the daughter nucleus is Beryllium-8 (Be-8), then what other particle is emitted?
   
   c) Nitrogen-14 (N-14) is bombarded by a particle. As a result, Carbon-14 (C-14) is created and a proton is emitted as a result. What is the original particle?

2. Uranium-235 is bombarded by a slow-moving neutron, and as a result of the weak nuclear force, the uranium splits into two smaller nuclei and emits three neutrons. If one of the smaller daughter nuclei is Tin-131 (Sn-131). What is the other daughter nucleus?

3. In a fusion reaction, two Helium-3 (He-3) nuclei slam into each other at very high speeds and fuse together to form an alpha particle. Two other identical particles are also emitted. What are these particles?

4. U-235 (235.044 u) is bombarded by a slow moving neutron (1.009 u). As a result, Xenon-140 (139.922 u) and Strontium-94 (93.915 u) are formed, and two neutrons are emitted. How much energy (in J) is liberated in this reaction? Answer to 4 digits.

5. In a fusion reaction, two deuterium nuclei (each of mass 2.014 u) fuse together and form Helium-3 (3.016 u) and a neutron (1.009 u). Determine the energy liberated from this reaction (in MeV). Answer to 3 digits.

**SOLUTIONS**

1. a) \( ^{27}_{13}Al + ^{4}_{2}He + ^{3}_{1}P + ^{30}_{14}Si \) Silicon-30
   
   b) \( ^{9}_{4}Be + ^{1}_{0}n + ^{8}_{4}Be + ^{2}_{1}H \) Deuterium (H-2)
   
   c) \( ^{14}_{7}N + ^{1}_{0}n \) A neutron

2. \( ^{235}_{92}U + ^{1}_{0}n + ^{13}_{6}C + ^{92}_{56}Mo + ^{3}_{0}n \) Molybdenum-102

3. \( ^{1}He + ^{1}He + ^{2}He + ^{2}C \) Two protons are emitted.

4. \( 11m = 0.198 \text{ u} = 3.2868 \times 10^{-26} \text{ kg} \quad LiE = 2.958 \times 10^{-11} \text{ J} \)

5. \( 11m = 0.003 \text{ u} = 4.98 \times 10^{-26} \text{ kg} \quad LiE = 4.482 \times 10^{-13} \text{ J} = 2.80 \text{ MeV} \)

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B. ELEMENTARY PARTICLE THEORY

We have discussed atomic theory, and then we have extended our knowledge down to the nucleus. Now, we are going to go further, looking into the structure of the nucleons themselves. By 1932, there were four "elementary" particles of matter known: the electron, the proton, the neutron, and the neutrino. In addition, there were the positron (antiparticle to the electron) and the photon of EMR.

However, with the use of high-energy particles (from cosmic rays and particle accelerators) and studying the resulting particle tracks, new particles were discovered.

Bl. Particle Tracks

In the early history of elementary particle physics, two different devices were used to investigate the nature of new particles created by reactions:

1. Cloud Chambers (C.T.R. Wilson, 1910)

In a cloud chamber, a gas is cooled to a temperature slightly below its usual condensation point (called "supercooled"). When charged particles enter the mixture, they act as condensation nuclei (much like dust particles in cloud formation) and tiny droplets of mist form around them. So, as the ions travel through the chamber, they leave tiny cloud trails (or tracks) that can be photographed and analyzed. However, neutral particles leave no tracks.

These tracks have distinctive shapes, especially when the cloud chamber is subjected to a vertical (perpendicular) magnetic field.

An alpha particle's track is broad, since it is relatively large and has many collisions with the gas atoms. It deflects in a magnetic field according to RHR #3, due to its positive charge. However, because it has a larger mass, it does not deflect very much - that is, its path is relatively straight.

A beta particle's track is thinner and it shows much more evidence of deflection (due its smaller mass). If it is a beta-negative particle (i.e. an electron), its deflection obeys LHR #3; if it is a beta-positive particle (i.e. a positron), it obeys RHR #3.

Neutral particles and photons leave no tracks. Their presence is inferred by subsequent decays.

Note: The particle tracks spiral, since they lose energy from the collisions with the gas molecules.

2. Bubble chambers (D.A. Glaser, 1952)

In a bubble chamber, a liquid is kept close to the boiling point. When an ion travels through it, it leaves a trail of bubbles (characteristic of boiling), much like the droplets for a cloud chamber.

Since bubble chambers use liquids (usually liquid hydrogen), which have a greater density of atoms than the cloud chamber, it more efficiently tracks the path and the interactions with the nuclei of the liquid.
B2. Particle Accelerators

Why High Energy Particles?

In order to investigate nucleus, the incident particles must have enough energy to overcome the strong nuclear force between the nucleons. The strong nuclear force is the largest known force in nature. As a result, the energies of the incident particles must be in the order of GeV or TeV.

These energies could be achieved naturally by cosmic rays, which are particles (mostly protons) that come to Earth from space due to solar flares or supernovae. The energies of cosmic rays can range from $10^2$ to $10^{14}$ MeV! However, most of these rays are absorbed by the Earth's atmosphere and so very few actually reach the Earth's surface.

In order to have a more reliable and more controllable source of high energy particles to be used in the lab, scientists created their own devices called particle accelerators.

Particle Accelerators

Many particle accelerators exist, including Van de Graaf generators, linear accelerators, and synchrotrons. You are invited to investigate these independently.

We will study one device in detail, since it is an excellent review for Physics 30.

Cyclotron (E.O. Lawrence, 1930)

A cyclotron is composed of two hollow chambers called "dees", that are filled with a magnetic field to contain the path of the protons, and they are separated by a gap which contains an alternating electric field for linear acceleration.

Protons are released from a source, located midway between the dees. The voltage from the AC source creates an electric field (from left to right) to accelerate the proton towards the right dee. When it enters the right dee, the magnetic field causes it move in uniform circular motion and thus contains the particle. When it leaves the right dee, the electric field has reversed in direction (due to the AC source) and so, the proton continues speeds up even more towards the left dee. When enters the left dee, it undergoes uniform circular motion but with a larger radius - due to its higher speed.

After each cycle, the proton gains more speed and forms circular paths of larger radii. Eventually, when the proton has reached the maximum speed (radius), it exits the cyclotron and hits a target atom.

The maximum energy of the emitted proton is limited by the strength of the magnetic field. As well, this device gets difficult when approaching relativistic speeds (since mass increases).

Note: Since the particles are constantly accelerating at very high rates, they will lose a lot of energy as high frequency EMR - called synchrotron radiation. Thus, certain particle accelerators (like synchrotrons) can be a good source of extremely high energy gamma radiation for experiments.
HOMEWORK (Particle Accelerators)  Ignore relativistic effects for #1 - 6

Both A and B

1. When Promethium-61 goes through alpha decay, it emits an alpha particle to the left and a gamma ray to the right. The gamma ray then decays into an electron and a positron. Sketch the tracks you would see in a bubble chamber for this interaction, if the magnetic field is into the page. Explain.

2. Two particles enter a perpendicular field with equal speeds. If particle 1 has a smaller radius, then how do their charge-to-mass ratios compare? Justify mathematically.

3. A cyclotron with a maximum radius of 25.0 cm accelerates protons in a 1.70 T perpendicular magnetic field. Determine the maximum kinetic energy of the protons (in MeV) when they leave the cyclotron.

4. The voltage across the dees in a cyclotron is 40.0 kV. If a proton enters the gap with a speed of $3.20 \times 10^6$ m/s, then at what speed does it enter the opposite dee?

B. 5. It is a well-known fact that the shorter the wavelength of the cosmic radiation, the better "resolution" of the analysis and thus the more information liberated from an experiment. Would it be better to have 20.0 MeV protons or 20.0 MeV alpha particles in the experiment? Explain, using calculations.

6. The voltage across the dees in a cyclotron is 50.0 kV. How many revolutions do the protons have to make to reach a speed of $8.00 \times 10^7$ m/s.
   (Hint: Find energy gained each revolution)

SOLUTIONS

1. Positive particles obey RHR #3, while negative particles obey LHR #3. The alpha particle is larger and more massive so it makes a thicker line and has a larger radius. The gamma ray photon makes no track in a bubble chamber, since it is neutral. But, we can infer its path based on the electron-positron pair that it produces.

2. Circular motion: $r_{\text{rot}} = \frac{mv}{qB}$  Inverse relationship

   Particle 1 has a larger q/m ratio.

3. $v = 4.072 \times 10^7$ m/s  ; $E_k = 8.65$ MeV

4. $E_{\text{kJ}} = 1.495 \times 10^{-14}$ J  ; $v = 4.23 \times 10^6$ m/s

5. **Proton:** $v = 6.19 \times 10^7$ m/s; $A_p = 6.41 \times 10^{-15}$ m; **Alphaparticle:** $v = 3.10 \times 10^7$ m/s; $A_\alpha = 3.21 \times 10^{-16}$ m; So, the alpha particles would have better resolution.

6. $E_{\text{kJ}} = 33.4$ MeV; Each half-revolution, the proton gains 50 keY; so, each revolution, the proton gains 100 keY; Thus, it needs 334 revolutions.
B3. PARTICLE CLASSIFICATION

Using particle accelerators (and cosmic rays) and particle tracks, hundreds of new subnuclear particles were discovered. Scientists struggled to find some underlying order in this "particle zoo" and thus, find a way to classify them in a meaningful way. They found that the most promising approach was to base their classification on the fundamental forces - listed from strongest to weakest: strong nuclear force, the electromagnetic force, the weak nuclear force, and the gravitational force.

There are three major categories of particles:

1. Leptons
   - these particles do not experience the strong nuclear force, but they do experience the weak nuclear force, the gravitational force, and the electromagnetic force (if they have charge)
   - leptons include the electron (e-) and neutrinos
   - leptons are believed to be true elementary particles with zero size and no internal structure

2. Hadrons
   - these particles do experience the strong nuclear force (but they also interact with the other fundamental forces, although the strong force is dominant in short distances)
   - these include the proton and neutron

3. Field Particles (or, Gauge Bosons)
   - these are the quanta of the different force fields and "carriers" (mediators) of these forces
   - field particles include the photon

Particles and Antiparticles

All of the particles listed above also have associated antiparticles, which are identical in every way but they have opposite charges (and other advanced properties). For example, the antiparticle of an electron (e-) is the positron (e+) and the antiparticle of a proton (p+) is an antiproton (p-). Some neutral particles have antiparticles (such as neutrinos), but some are their own antiparticles (such as photons).

Antiparticles are produced in nuclear reactions when there is sufficient energy available, and they do not live very long in the presence of other matter. For example, a positron is stable in itself, but when it encounters and electron, they annihilate each other and gamma radiation (for example) is produced. All other particle-antiparticle interactions lead to annihilation as well.

When you work with annihilation, you must be certain to account for the mass defect, since all of the mass is converted to energy.

For example, when an electron and a positron collide and annihilate each other, they create two gamma photons. Using conservation of energy, it follows that:

\[ E_{Ti} = E_{Tf} \]

\[ E_{t} > m(e-) + E_{t} m(e+) = 2E_{r} \]

\[ 2E_{\gamma} (e-) = 2E_{\gamma} \]

Note: The initial kinetic energies are ignored here.
Quarks

Recall that leptons are considered to be true elementary particles, which means they have no size and have no internal structure. However, there are hundreds of hadrons and some scientists were unwilling to believe that these were all elementary particles. In fact, in 1963, M. Gell-Mann and G. Zweig independently proposed that all hadrons were composed of elementary particles called quarks.

Types of Quarks

To account for the internal structure of neutrons and protons, scientists needed two different kinds of quarks: up and down. Their properties are listed below:

<table>
<thead>
<tr>
<th>Quark name</th>
<th>Symbol</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up</td>
<td>u</td>
<td>+(\frac{2}{3}e)</td>
</tr>
<tr>
<td>Down</td>
<td>d</td>
<td>-(\frac{1}{3}e)</td>
</tr>
</tbody>
</table>

Note:
- Other quarks: strange (s), charmed (c), bottom or beauty (b), and top or truth (t)
- All quarks have corresponding antiparticles, which have opposite charge.

Quarks and Hadrons

The quarks combine in pairs and triplets to form all known hadrons. For example, the proton is composed of 2 up quarks and one down quark \(uud\), while the neutron is composed of 1 up quark and 2 down quarks (i.e. \(udd\)).

Beta Decay Revisited

We can now look at beta decay in more detail, with our new knowledge of subnuclear particles. Recall that in beta-negative decay, the neutron decays into a proton and emits a beta particle and an anti-neutrino as a result. Since the two emitted particles are leptons, then this reaction must occur because of the weak nuclear force. This can be shown as

\[
\text{udd} \rightarrow uud + jr + u
\]

In beta-positive decay, the proton decays into a neutron and emits a positron and a neutrino as a result. Again, since the two emitted particles are leptons, this reaction must occur because of the weak nuclear force.
force. This can be shown as

\[ u_{ud} \ u_{dd} \ + \ jF \ + \ u \]
Homework (Particle Classification)

1. An electron neutrino and an electron antineutrino collide and when they do, they annihilate each other and create two identical photons. Determine the frequency of each photon emitted.

2. A gamma ray photon decays into an electron and a positron inside of a perpendicular magnetic field directed out of the page. Both daughter particles have an energy of 80 keV.
   a) Sketch and describe their tracks, as seen in a bubble chamber. Explain.
   b) If a scientist measured the radius of curvature to be 3.50 mm for both particles, then determine the strength of the magnetic field. Ignore relativistic effects.
   c) Determine the wavelength of the gamma ray photon. Don’t forget to include the mass-energies of the two particles produced, as provided on the formula sheet.

3. Determine the overall charge of the following combinations of quarks and antiquarks.
   Note: A strange quark (s) has the same charge as a down quark.
   a) s d d
   b) u s s
   c) u d
   d) u u d

4. In positron-emission tomography (PET), fluorine-18 is used to produce positrons.
   a) Provide a complete balanced equation for this decay.
   b) Explain the role of quarks in this decay.
   c) When the positron is created inside of the human body, it collides with an electron (at rest) in nearby tissue and they are mutually annihilated. Two gamma ray photons are created. If each gamma photon has a frequency of $1.30 \times 10^{20}$ Hz, then determine the kinetic energy of the positron before the collision. Answer in keV.

Solutions

1. Cons of mass-energy: For each particle, $E_{Li/m} = 50$ eV
   $2E_{Li/m} = 2E_{photon}$; $j = 1.2 \times 10^{16}$ Hz

2. a) The gamma photon does not leave a path, since it is neutral.
   Both particles go into uniform circular motion. The positron goes clockwise (using RHR #3), while the electron goes counterclockwise (using LHR #3).
   b) $v = 1.676 \times 10^8$ m/s; $B = 0.273$ T
   c) $Er = 2E_{k+}$ $2E_{L/m} = 2(80 \times 10^3$ eV) + $2(0.511 \times 10^6$ eV) = $1.182$ MeV; $A = 1.05$ pm

3. a) $-1e$
   b) Neutral
   c) $+1e$
   d) $-1e$

4. a) $i \cdot F$ $i \cdot J$ $j \cdot o$ $g \cdot u$ (+ r)
   b) An up quark changes to a down quark, and releases a positron and a neutrino
   I.e. $u$ $d$ $d$ $u$ $d$ $d$ $+ B^+$ $u$
   c) $E_m = 0.511$ MeV; $E_e = 0.538$ MeV; $E_{k(e+)} + 2E_{Li/m} = 2E_r$; $E_k = 54.4$ keV

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